Features for Landcover Classification of Fully Polarimetric SAR Data

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ABSTRACT

We have previously shown that Stokes eigenvectors can be numerically extracted from the Kennaugh(Stokes) matrices of both single-look and multilook fully polarimetric SIR-C data. The extracted orientation and ellipticity parameters of the Stokes eigenvector were found to be related to the Huynen orientation and helicity parameters for single-look fully polarimetric SIR-C data. We formally show in this paper that these two parameters, which diagonalize the Sinclair matrices of the single-look data, belong to a set of parameters which diagonalize the Kennaugh matrices of single-look data. Along with the cross sections $|S_{vv}|^2$, $|S_{hv}|^2$, $|S_{hh}|^2$ and the Span, the eigenvalues of the Kennaugh matrix and the covariance matrix are used as input features in the development of a neural net landcover classifier for SIR-C data.

1. INTRODUCTION

Geaga\textsuperscript{1–3} in a series of papers has reported on the numerical eigenanalysis of the Kennaugh(Stokes) matrices of fully polarimetric single-look and multi-look SIR-C and ISAR range data. We now present a formal derivation of the diagonalization of the Kennaugh matrix when the Sinclair matrix is available(single-look SIR-C data and ISAR range data). The four consecutive similarity transformations which diagonalize the Kennaugh matrix are outlined. The diagonal elements of the resulting diagonalized matrix are the eigenvalues of the Kennaugh matrix. The product of the similarity transformations provides the eigenvectors of the Kennaugh matrix. Three of the similarity transformation parameters are equivalent to three of the Huynen\textsuperscript{4} decomposition parameters, namely the orientation, helicity and bounce angles. The analytic formulations of the eigenvalues found for the Kennaugh matrices of single-look SIR-C data suggest a basis for interpreting the eigenvalues of multilook data, where the eigenvalues can only be determined numerically, using the Jacobi algorithm.\textsuperscript{5} These eigenvalues are shown to have units of cross section. The eigenvalues of the covariance matrix are discussed and have units of cross section as well. Along with the cross sections $|S_{vv}|^2$, $|S_{hv}|^2$, $|S_{hh}|^2$ and the Span, the eigenvalues of the Kennaugh matrix and the covariance matrix are used as input features in prototyping a neural net landcover classifier for multi-look SIR-C data. The neural net classifier architecture is discussed. Initial classifier results on multi-look SIR-C data are presented. Decomposition of the Sinclair matrix into contributions from a sphere, a rotated dipole and a helix is discussed promising additional features for use in developing classifiers for single-look SIR-C data. A general framework is developed for choosing fully polarimetric single-look and multi-look input features for use in landcover classifiers for fully polarimetric SAR.

2. THE KENNAUGH MATRIX, SINGLE-LOOK SIR-C DATA

A derivation of the Kennaugh matrix in terms of the elements of the Sinclair matrix was given by Geaga.\textsuperscript{2} The matrix elements of the Kennaugh matrix are given below in Equation 1 to provide consistent notation throughout this discussion. The Kennaugh matrix is a real symmetric matrix and can be diagonalized using a similarity transformation. The elements of the diagonalized matrix are the eigenvalues of the Kennaugh matrix and the columns of the similarity transformation matrix are the eigenvectors. For a real symmetric matrix, the eigenvectors and eigenvalues are all real.

\[
M_{11} = \frac{1}{4} (S_{ev}^* S_{ev} + 2 S_{hv}^* S_{hv} + S_{hh}^* S_{hh})
\]

\[
M_{12} = \frac{1}{4} (S_{ev}^* S_{ev} - S_{hh}^* S_{hh})
\]
\[ M_{13} = \frac{1}{2} \Re(S_{hv}^*S_{vv}) + \frac{1}{2} \Re(S_{hh}^*S_{hv}) \]
\[ M_{14} = -\frac{1}{2} \Im(S_{hv}^*S_{vv}) - \frac{1}{2} \Im(S_{hh}^*S_{hv}) \]
\[ M_{22} = \frac{1}{4}(S_{vv}^*S_{vv} - 2S_{hv}^*S_{hv} + S_{hh}^*S_{hh}) \]
\[ M_{23} = -\frac{1}{2} \Re(S_{hv}^*S_{vv}) - \frac{1}{2} \Re(S_{hh}^*S_{hv}) \]
\[ M_{24} = -\frac{1}{2} \Im(S_{hv}^*S_{vv}) \]
\[ M_{33} = \frac{1}{2} \Re(S_{hh}^*S_{vv}) + \frac{1}{2} S_{hv}^*S_{hv} \]
\[ M_{34} = -\frac{1}{2} \Im(S_{hh}^*S_{vv}) \]
\[ M_{44} = \frac{1}{2} S_{hv}^*S_{hv} - \frac{1}{2} \Re(S_{hh}^*S_{vv}) \]

The matrix element \( M_{11} \) is equal to \( \frac{\text{Span}}{4} \) where

\[ \text{Span} = ||S_{vv}||^2 + 2||S_{hv}||^2 + ||S_{hh}||^2 \]

or the total cross section.

### 2.1 Diagonalizing the Kenough Matrix

The Kenough matrix of a single-look SIR-C pixel can be analytically diagonalized using the following similarity transformations

\[ M''' = R_{\phi}^T R_{\nu}^T R_{\tau}^T R_{\phi} M R_{\nu} R_{\tau} R_{\phi}. \]

The elements of the diagonal matrix \( M''' \) are the eigenvalues of the Kenough matrix. Baird\(^6\) represents the \( \phi \), \( \tau \) and \( \nu \) transformations as

\[ R_{\phi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\phi & -\sin 2\phi & 0 \\ 0 & \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]
\[ R_{\tau} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\tau & 0 & -\sin 2\tau \\ 0 & 0 & 1 & 0 \\ 0 & \sin 2\tau & 0 & \cos 2\tau \end{pmatrix} \]
\[ R_{\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos 2\nu & -\sin 2\nu \\ 0 & 0 & \sin 2\nu & \cos 2\nu \end{pmatrix} \]
where $\phi$, $\tau$ and $\nu$ are the Huynen orientation, helicity and bounce(skip) angles respectively. The $\alpha$ transformation is

$$R_{\alpha} = \begin{pmatrix} \cos 2\alpha & -\sin 2\alpha & 0 & 0 \\ \sin 2\alpha & \cos 2\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (7)$$

The $\phi$ parameter can be determined by using the condition prescribed by Huynen$^4$ that $M'_{13} = 0$ where

$$M' = R_{\phi}^T M R_{\phi}. \quad (8)$$

This gives

$$\phi = \tan^{-1} \left( \frac{-M_{12} + \sqrt{M_{12}^2 + M_{13}^2}}{M_{13}} \right). \quad (9)$$

The same transformation is achieved$^3$ through the following similarity transformation on the Sinclair matrix

$$S' = U^T S U \quad (10)$$

where

$$U = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}. \quad (11)$$

The parameter $\phi$ in Equation 9 is utilized. The parameter $\tau$ can be determined using the condition $M''_{44} = 0$ where

$$M'' = R_{\tau}^T M' R_{\tau}. \quad (12)$$

This gives

$$\tau = \frac{1}{2} \tan^{-1} \left( \frac{M'_{14}}{M_{12}} \right). \quad (13)$$

The same transformation is again achieved$^3$ by the following similarity transformation on the the Sinclair matrix $S'$

$$S'' = V^T S' V \quad (14)$$

where

$$V = \begin{pmatrix} \cos \tau & -i \sin \tau \\ -i \sin \tau & \cos \tau \end{pmatrix}. \quad (15)$$

The parameter $\tau$ determined in Equation 13 is again used. The resulting matrix $S''$ is diagonal. Because $S_{hv} = 0$, $M''_{23} = M''_{24} = 0$ results as follows from Equation 1. This gives
\[
M'' = \begin{pmatrix}
M_{11}'' & M_{12}'' & 0 & 0 \\
M_{12}'' & M_{22}'' & 0 & 0 \\
0 & 0 & M_{33}'' & M_{34}'' \\
0 & 0 & M_{34}'' & M_{44}''
\end{pmatrix}.
\]

(16)

It also follows from Equation 1 that \(M_{22}'' = M_{11}''\) and \(M_{44}'' = -M_{33}''\). The parameters \(\phi\) and \(\tau\) are the parameters that diagonalize the Sinclair matrix. These parameters are the Huynen orientation and helicity Euler parameters. It is shown below that these are also the orientation and ellipticity parameters of the Stokes eigenvector of the Kennaugh matrix. The span is conserved by the preceding transformations, i.e. \(M_{11}''' = M_{11}''\).

The parameter \(\nu\) is determined using the condition \(M_{34}''' = 0\) where
\[
M''' = R_{\nu}^T M'' R_{\nu}.
\]

(17)

This gives
\[
\nu = \frac{1}{4} \tan^{-1} \left( \frac{2M_{34}''}{M_{33}'' - M_{44}''} \right).
\]

(18)

This transformation gives \(M_{11}''' = M_{11}''\), \(M_{12}''' = M_{12}''\) and \(M_{22}''' = M_{22}''\). Additionally, we find using Equations 18 and 1 that
\[
\nu = \frac{1}{4} (\phi''_{vv} - \phi''_{hh}).
\]

(19)

This is the Huynen bounce(skip) angle.

The parameter \(\alpha\) can be determined using the condition \(M_{12}''' = 0\) where
\[
M''' = R_{\alpha}^T M''' R_{\alpha}.
\]

(20)

We get \(\alpha = 22.5^\circ\) when \(M_{12}''' = M_{12}'' > 0\). Using Equation 1, this condition is equivalent to \(||S_{vv}'''|| > ||S_{hh}'''||\). Alternately, we get \(\alpha = -22.5^\circ\) if \(M_{12}' < 0\). Analysis of the single-look SIR-C data and the ISAR turntable data indicates that only \(M_{12}'' > 0\) is physically realized. Throughout this discussion, we will assume that \(M_{12}'' > 0\), hence that \(\alpha = 22.5^\circ\).

2.2 Kennaugh Matrix Stokes Eigenvector

The first eigenvector which is extracted from the similarity transformations is
\[
F_0 = R_{\phi} R_{\tau} R_{\nu} R_{\alpha} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{\cos \phi}{\sqrt{2}} \cos 2\tau \\ \frac{\sin \phi}{\sqrt{2}} \cos 2\tau \\ \frac{\sin \phi}{\sqrt{2}} \sin 2\tau \end{pmatrix}.
\]

(21)

This has the form of a Stokes vector \(\mathbf{s}^T\) where \(\psi = \phi\) and \(\chi = \tau\). The parameters which diagonalize the Sinclair matrix are identical to the parameters found in the Stokes eigenvector of the Kennaugh matrix. This Stokes vector is fully polarized. For the \(\alpha = -22.5^\circ\) solution, the Stokes eigenvector if realized would be
\[ F_0 = \left( \begin{array}{c}
-\frac{1}{\sqrt{2}} \\
\cos 2\phi \cos 2\tau \\
\sin 2\phi \cos 2\tau \\
\sin 2\tau
\end{array} \right) = \left( \begin{array}{c}
\frac{1}{\sqrt{2}} \\
\cos 2\phi_1 \cos 2\tau_1 \\
\sin 2\phi_1 \cos 2\tau_1 \\
\sqrt{2}
\end{array} \right) \] (22)

where \( \phi_1 = \phi + \frac{\pi}{2} \) and \( \tau_1 = -\tau \).

### 2.3 Quaternions

The three remaining eigenvectors are

\[ F_1 = \left( \begin{array}{c}
-\frac{1}{\sqrt{2}} \\
\cos 2\phi \cos 2\tau \\
\sin 2\phi \cos 2\tau \\
\sin 2\tau
\end{array} \right) \] (23)

\[ F_2 = \left( \begin{array}{c}
0 \\
-\sin 2\phi \cos 2\nu - \cos 2\phi \sin 2\tau \sin 2\nu \\
\cos 2\phi \cos 2\nu - \sin 2\phi \sin 2\tau \sin 2\nu \\
\cos 2\tau \sin 2\nu
\end{array} \right) \] (24)

\[ F_3 = \left( \begin{array}{c}
0 \\
\sin 2\phi \sin 2\nu - \cos 2\phi \sin 2\tau \cos 2\nu \\
-\cos 2\phi \sin 2\nu - \sin 2\phi \sin 2\tau \cos 2\nu \\
\cos 2\tau \cos 2\nu
\end{array} \right) \] (25)

These have a quaternion form

\[ q = (\cos \frac{\rho}{2}, \hat{v} \sin \frac{\rho}{2}) \] (26)

where this quaternion represents a rotation of \( \rho \) about the unit vector direction \( \hat{v} \). This gives \( \rho_1 = 270^\circ \) and \( \rho_2 \) and \( \rho_3 \) are equal to \( 180^\circ \). The unit vectors are

\[ v_1 = \left( \begin{array}{c}
\cos 2\phi \cos 2\tau \\
\sin 2\phi \cos 2\tau \\
\sin 2\tau
\end{array} \right) \] (27)

\[ v_2 = \left( \begin{array}{c}
-\sin 2\phi \cos 2\nu - \cos 2\phi \sin 2\tau \sin 2\nu \\
\cos 2\phi \cos 2\nu - \sin 2\phi \sin 2\tau \sin 2\nu \\
\cos 2\tau \sin 2\nu
\end{array} \right) \] (28)

\[ v_3 = \left( \begin{array}{c}
\sin 2\phi \sin 2\nu - \cos 2\phi \sin 2\tau \cos 2\nu \\
-\cos 2\phi \sin 2\nu - \sin 2\phi \sin 2\tau \cos 2\nu \\
\cos 2\tau \cos 2\nu
\end{array} \right) \] (29)

These unit vectors are orthonormal where \( v_i \cdot v_j = 0 \) for \( i \neq j \) and \( v_1 \cdot v_1 = 1 \). Numerical analysis by Geaga\(^3\) of the single-look SIR-C data and radar turntable data reflected this.
2.4 Kennaugh Pseudo-eigenvalue Equation

Kostinski and Boerner\(^8\) refer to the equation

\[ S \begin{pmatrix} E_v \\ E_h \end{pmatrix} = \lambda \begin{pmatrix} E_v^* \\ E_h^* \end{pmatrix}, \quad (30) \]

where \( S \) is the Sinclair matrix, as the Kennaugh “pseudo-eigenvalue equation”. We now derive the eigenvectors and eigenvalues. The transformed and diagonalized Sinclair matrix can be expressed as

\[ S'' = R^T S R \quad (31) \]

where \( R = UV \). The eigenvectors of the diagonalized matrix \( S'' \) can be arbitrarily set as \([1, 0]\) and \([0, 1]\) for \([E_v'', E_h'']\). We then have the two eigenvalue equations

\[ S'' \begin{pmatrix} 1 \\ 0 \end{pmatrix} = S''_{vv} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (32) \]

and

\[ S'' \begin{pmatrix} 0 \\ 1 \end{pmatrix} = S''_{hh} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (33) \]

These are both of the form

\[ S'' x'' = \lambda x''. \quad (34) \]

Multiplying Equation 34 on the left on both sides by \((R^T)^{-1}\), we get

\[ (R^T)^{-1} S'' R^{-1} R x'' = \lambda (R^T)^{-1} x''. \quad (35) \]

We can substitute the identity \( S = (R^T)^{-1} S'' R^{-1} \) in Equation 35. For the eigenvector \( x'' = [1, 0] \), we get

\[ R x'' = \begin{pmatrix} \cos \phi \cos \tau + i \sin \phi \sin \tau \\ \sin \phi \cos \tau - i \cos \phi \sin \tau \end{pmatrix} \quad (36) \]

and

\[ (R^T)^{-1} x'' = \begin{pmatrix} \cos \phi \cos \tau - i \sin \phi \sin \tau \\ \sin \phi \cos \tau + i \cos \phi \sin \tau \end{pmatrix}. \quad (37) \]

The eigenvector in Equation 36

\[ \begin{pmatrix} E_v \\ E_h \end{pmatrix} = \begin{pmatrix} \cos \phi \cos \tau + i \sin \phi \sin \tau \\ \sin \phi \cos \tau - i \cos \phi \sin \tau \end{pmatrix}. \quad (38) \]

satisfies Equation 30. The Stokes vector for this pseudo-eigenvector is

\[ F = \begin{pmatrix} \frac{\|E_v\|^2 + \|E_h\|^2}{\|E_v\|^2 - \|E_h\|^2} \\ \frac{2 \text{Re}(E_v E_h^*)}{\|E_v\|^2 - \|E_h\|^2} \\ \frac{2 \text{Im}(E_v E_h^*)}{\|E_v\|^2 - \|E_h\|^2} \end{pmatrix} = \begin{pmatrix} 1 \\ \cos 2\phi \cos 2\tau \\ \sin 2\phi \cos 2\tau \end{pmatrix}. \quad (39) \]
We can normalize this Stokes vector by multiplying by $\frac{1}{\sqrt{2}}$ and this results in the Stokes eigenvector of the Kennaugh matrix (Equation 21). For Equation 33, we get

$$F_{\frac{1\sqrt{2}}{2}} = \begin{pmatrix}
1 \\
-\cos 2\phi \cos 2\tau \\
-\sin 2\phi \cos 2\tau \\
-\sin 2\tau
\end{pmatrix}.$$  \hspace{1cm} (40)

which is the Stokes eigenvector of the Kennaugh matrix when $M_{12}^{"} < 0$. The data indicates that this solution is never physically realized. Additionally, we have $F = -\sqrt{2}F_{1}$ from Equations 23 and 40.

The parameters $\phi$ and $\tau$ which diagonalize the Sinclair matrix are identical to the orientation and ellipticity parameters of the Stokes eigenvector of the Kennaugh matrix. The curious $\sqrt{2}$ factor reported by Geaga\textsuperscript{3} resulted from having ignored the $\sqrt{2}$ factor from $\alpha$.

2.5 Kennaugh matrix eigenvalues

The eigenvalues of the Kennaugh matrix are the diagonal elements of the diagonal matrix $M^{"}$ in Equation 3. These are

$$M_{11}^{"} = \frac{\|S_{vv}\|^2}{2} \hspace{1cm} (41)$$

$$M_{22}^{"} = \frac{\|S_{hh}\|^2}{2} \hspace{1cm} (42)$$

$$M_{33}^{"} = \frac{\|S_{vv}\|\|S_{hh}\|}{2} \hspace{1cm} (43)$$

$$M_{44}^{"} = -M_{33}^{"}. \hspace{1cm} (44)$$

It should be noted that these eigenvalues have units of cross section. There are three positive eigenvalues and one negative. It is easy to see that $M_{11}^{"} \geq M_{33}^{"} \geq M_{22}^{"}$. The numerically extracted eigenvalues for the multilook Kennaugh matrices analyzed and reported by Geaga\textsuperscript{1–3} also had three positive eigenvalues and one negative eigenvalue. However, the sum of the mid-valued eigenvalue and the negative eigenvalue never added up to zero and was always positive. The multilook eigenvalues are ordered in descending order of magnitude as $K_{1}, K_{2}, K_{3}$ and $K_{4}$ in our discussion of classifier features below. Equations 41 to 44 suggest the physical content in these eigenvalues.

3. COVARIANCE MATRIX

The Kennaugh and Mueller matrices are widely used power representations of the scattering properties of a target. Another widely used representation is the covariance matrix. The scattering vector or covariance vector $\vec{k}_{c}$ is a vectorized version of the Sinclair matrix

$$\vec{k}_{c} = \begin{pmatrix} S_{vv} \\ \sqrt{2}S_{hv} \\ S_{hh} \end{pmatrix}. \hspace{1cm} (45)$$

The covariance matrix is

$$\vec{k}_{c} \vec{k}_{c}^{\dagger} = \begin{pmatrix} S_{vv}S_{vv}^{*} & \sqrt{2}S_{vv}S_{hv}^{*} & S_{vv}S_{hh}^{*} \\ \sqrt{2}S_{hv}S_{vv}^{*} & 2S_{hv}S_{hv}^{*} & \sqrt{2}S_{hv}S_{hh}^{*} \\ S_{hh}S_{vv}^{*} & \sqrt{2}S_{hh}S_{hv}^{*} & S_{hh}S_{hh}^{*} \end{pmatrix}. \hspace{1cm} (46)$$
where $\vec{k}_c^\dagger$ is the adjoint of $\vec{k}_c$. The covariance matrix is a 3x3 hermitian matrix and its three eigenvalues are real. For single-look SIR-C data, it can be analytically shown that two of its eigenvalues are zero and the one non-zero eigenvalue is equal to the Span. For multi-look SIR-C data, the eigenvalues can be be determined using Cardano’s method. Preliminary results show that the largest eigenvalue for the multilook data is equivalent to the average span (over 4 looks). The two remaining eigenvalues are about 20 – 25% of the Span in magnitude, but of opposite sign. The sum of these two eigenvalues is always found to be positive. The eigenvalues of the covariance matrix have units of cross section, as with the Kennaugh matrix eigenvalues. The eigenvalues are ordered in descending order of magnitude as $C_1, C_2$ and $C_3$ in our discussion of classifier features below.

4. CLASSIFIER FEATURES

The 10 classifier input features used for the prototype multi-look neural net classifier are:

1. $\ll ||S_{vv}||^2 \gg$
2. $\ll ||S_{hh}||^2 \gg$
3. $\ll ||S_{hv}||^2 \gg$
4. $\ll \text{Span} \gg$
5. $K_1$
6. $K_3$
7. $K_2 + K_4$
8. $\frac{||K_2|| + ||K_4||}{2}$
9. $C_2 + C_3$
10. $\frac{||C_2|| + ||C_3||}{2}$

where the averages are over 4 looks for the multilook data. It should be noted that these features all have the same units of cross section. The features being considered for the single-look classifier are

1. $||S_{vv}||^2$
2. $||S_{hh}||^2$
3. $||S_{hv}||^2$
4. $\text{Span}$
5. $||S_{vv'}||^2$
6. $||S_{hh'}||^2$.

Additional candidate decomposition parameters for single-look data are discussed in the next section.
5. KROGAGER DECOMPOSITION

Krogager\textsuperscript{10} prescribed a scattering matrix decomposition which resolves a Sinclair matrix into three scatterer types: sphere, rotated diplane and a left or right helix. The decomposition is expressed as

\[ S = e^{i\phi} \left( e^{i\phi} k_s S_s + k_d S_d + k_h S_h \right) \]  

where \( S \) is the Sinclair matrix given by

\[ S = \begin{pmatrix} S_{vv} & S_{hv} \\ S_{hv} & S_{hh} \end{pmatrix} \]  

and the normalized Sinclair matrices of the sphere, rotated diplane, right helix and left helix are:

\[ S_s = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]  

\[ S_d = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{pmatrix} \]  

\[ S_{hr} = \frac{e^{i2\theta}}{2} \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} \]  

\[ S_{hl} = \frac{e^{-i2\theta}}{2} \begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix} \]  

The choice of the right or left helix is determined from the Sinclair matrix. Representing the total scattering parameter as

\[ k_{tot} = k_s + k_d + k_h \]  

we can consider the relative contributions of the scattering mechanisms \( m_{i=s,d,h} \) where

\[ m_i = \frac{k_i}{k_{tot}} \]  

The parameters \( k_s, k_d, k_h, \theta \) and \( \phi \) can be determined by transforming Equation 47 from the linear polarization basis to the circular polarization basis. This transformation is done by (Ruck\textsuperscript{11})

\[ \begin{pmatrix} S_{RR} \\ S_{LR} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \begin{pmatrix} S_{vv} & S_{hv} \\ S_{hv} & S_{hh} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} . \]  

This transformation gives

\[ S_{RR} = -iS_{hv} + \frac{1}{2} (S_{vv} - S_{hh}) \]  

\[ S_{RL} = \frac{1}{2} (S_{vv} + S_{hh}) \]  

\[ S_{LL} = iS_{hv} + \frac{1}{2} (S_{vv} - S_{hh}) . \]  

The parameter \( k_s \) is determined as

\[ k_s = \| S_{RL} \|. \]
For the right helix case ($\|S_{RR}\| \geq \|S_{LL}\|$)

\[
k_d = \|S_{LL}\|
\]
\[
k_h = \|S_{RR}\| - \|S_{LL}\|.
\] (58)

For the left helix case ($\|S_{LL}\| > \|S_{RR}\|$)

\[
k_d = \|S_{RR}\|
\]
\[
k_h = \|S_{LL}\| - \|S_{RR}\|.
\] (59)

The parameters $k_s$, $k_d$ and $k_h$ are real and positive. The diplane rotation parameter $\theta$ is determined as

\[
\theta = \frac{\phi_{RR} - \phi_{LL}}{4}.
\] (60)

The phase parameters are determined as

\[
\phi_s = \phi_{RL} - \frac{\phi_{RR} + \phi_{LL}}{2}
\]
\[
\phi = \frac{\phi_{RR} + \phi_{LL}}{2}.
\] (61)

$\phi$ is an absolute phase and is never considered. $\phi_s$ is the phase shift of the sphere contribution to the Sinclair matrix. Physically, the contributions from the sphere, rotated diplane and helix can represent the complex combination of scatterers present within the same resolution cell. Hong and Wdowinski\textsuperscript{12} have recently reported on dihedral and volume scattering behaviour in cross-polarimetric radar. They report that common vegetation scattering theories indicate that short wavelength SAR observations measure mainly vegetation canopies and reflect only volume scattering. However, short-wavelength and cross-pol observations from the Everglades wetlands in south Florida suggest that a significant portion of the SAR signal scatters from the surface and not only from the upper portion of the vegetation. The simplest multi-bounce scattering mechanism that generates cross-pol signal is a rotated dihedral. We will investigate the range of vegetation (farmland crops to forest woodland) in our single-look SIR-C data collection with these contributions in mind. Additionally, we will use this decomposition to investigate scattering mechanism differences from the bright and dark parts of the ocean discussed by Geaga\textsuperscript{3} among many other things.

6. NEURAL NET CLASSIFIER

The architecture we utilize in our prototype neural network is a feedforward architecture consisting of three layers, an input layer with ten(10) nodes, a hidden layer with ten(10) nodes and an output layer with five(5) nodes. Two of these networks were used as will be described below. The hidden layer and output layer nodes are implemented using sigmoidal activation functions. This feedforward network architecture is reported in\textsuperscript{13} and shown in Figure 1. It is a network for learning discriminants for patterns. This three layer neural network approach specifically differs from statistical classifiers in that the decision spaces can be both non-linear and disjoint. The $w_{kj}$ and $w_{ji}$ are weights which are learned from training the network, using ground truth data. The backpropagation learning algorithm as implemented by Pao\textsuperscript{14} is used for training the network.

6.1 Training Data

Training data was collected at the 6 ranges of Span listed below.

1. level-a (.001 to .01) classes 1,2
2. level-b (.01 to .02) classes 3,4
3. level-c (.02 to .1) class 5
4. level-d (.1 to .2) class 6
5. level-e (.2 to .3) classes 7,8
6. level-f (.3 up) classes 9,10

Training data for levels a-c was collected from the areas indicated in Figure 2a which is a false color image from a multi-look SIR-C scene in Death Valley, CA. Training data for levels d-f was collected from the areas indicated in Figure 2b. The number of classes readily discernible from the means and standard deviations of the 10 multi-look features of the data collected at the various \( \text{Span} \) ranges is shown in the list. Two classes were easily discernable in levels a, b, e and d. Work is in progress to develop methods for detecting additional classes. The partitioning into the six levels resulted in a total of 10 classes. An attempt to build a net with 10 output classes was unsuccessful leading to a preliminary conclusion that the cross section signal strengths need to be compartmented. A network for levels a-c with 5 output classes and a network for levels d-f with 5 output classes were successfully trained. The resulting classification of the SIR-C image is discussed in the next section. A threshold of 0.1 in \( \text{Span} \) was used to choose which network was used during the classification.

### 6.2 Classifier Results

The convergence of the solution for the first network (levels a-b) over 50000 training iterations is shown in Figure 3. Each network was presented with 500 training samples. The result of applying the two trained networks to a multi-look SIR-C Death Valley scene (around the Furnace Creek area) is shown in Figure 4. The class color map is shown on the top right of the class map image. Shown on the right hand part is a Landsat 5 true color image of the same scene. The airstrip just northwest of Furnace Creek is easily discernable as the yellow strip (lowest \( \text{Span} \) level) in the class map. The structures in Furnace Creek are seen in red (strongest scatterers) in the class map. The shrub vegetation southwest of Furnace Creek are also easily discernable in the class map.

### 7. CONCLUSION

The class map shown in Figure 4 shows that mapping polarimetric features to landcover classes is possible. A lot of work needs to be done in developing strategies for determining classes available for mapping in a particular dataset. Work is currently being pursued in developing discriminant algorithms for different types of vegetation. The different features identified can also be used to help develop scattering models for the different types of landcover types. Work is also underway in considering use of additional hidden layers in the neural net architecture employed. Along with the SIR-C data collected, these techniques could also be applied to fully polarimetric TerraSAR-X and Radarsat2 data.
Figure 2. Training areas

Figure 3. Network Convergence
REFERENCES


